

# Energy Efficiency in Cache Enabled Small Cell Networks With Adaptive User Clustering

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## Abstract

The exponential increase in mobile data traffic forces network operators to deal with a capacity shortage. One of the most promising technologies for 5G networks is proactive caching. Using a network of cache enabled small cells, traffic during peak hours can be reduced through proactively caching the content that is most probable to be requested. Studying the users behavior and caching files at base stations accordingly, can offload the backhaul traffic and improve the network throughput. We explore a new caching framework, in which, the users are clustered according to their content popularity. The caching is then done based on the mean content popularity in each cluster. In order to achieve an efficient clustering of the users, we use a statistical model selection criterion, namely the *Akaike information criterion*. We derive a closed form expression of the hit probability, which will be optimized with respect to the fractions of small base stations associated to each cluster. We then investigate the Energy Efficiency of the proposed caching framework. We derive a closed form expression of the energy efficiency, which will be optimized by defining the optimal density of active small base stations. We then provide a small base station allocation algorithm in order to associate each individual base station with a given cluster. This algorithm aims at caching in each small base station the files that are most likely to be requested within their direct neighborhood. Coupled with channel inversion power control, this optimization will improve the energy efficiency and cache hit probability of the network. Numerical results show that the clustering scheme enable to considerably improve the cache hit probability. We also show that optimizing the allocation of the small base stations results in improving of the energy efficiency and hit probability in the network.

Part of this work has been presented at the 17th IEEE International workshop on Signal Processing advances in Wireless Communications [1].

## I. INTRODUCTION

The global mobile traffic is expected to increase exponentially in the coming years. This is mainly due to the wide spread of wireless devices and the emergence of video streaming as one of the main contributors in mobile data traffic. In order to cope with the explosive throughput demand and the requirement for 5G networks, a number of new technologies are considered namely, heterogeneous networks and massive MIMO. Network Densification through the deployment of Small Base stations was proposed as an effective mean to provide higher throughput and to offload an important amount of traffic from the macro base stations. However, this will require the deployment of a costly infrastructure and a considerable increase in the back-haul link capacity. Recently, Information centric networks are emerging as an efficient technology to offload traffic and reduce the strains on the backhaul. In fact, since a substantial part of the mobile traffic is due to several duplicate requests of a limited popular contents, proactively caching these files in the edge of the network will result in enhancing user experience while reducing the needed back-haul link capacity. In addition to satisfying the increasing demand, sustainable development is also a major requirements for 5G networks. In fact, the urgent need to reduce global green gazes emission, made the energy efficiency one of the major key performance indicators for future networks. Thanks to the improvement of memory devices, proactive caching offers a very practical and energy efficient alternative to network densification. The idea of Caching popular content on the edge of the network is gaining momentum [2] [3]. In [4], a joint routing and caching problem in small cell networks was considered, taking into account both the constrained storage and transmission bandwidth capacities of the small base stations. The authors used approximation algorithms with performance grantees in order to derive a solution that maximizes the content requests that are satisfied by the SBSs. In [5], an information-theoretic formulation of the caching problem was considered. The authors proposed a coded schemes that enables a considerable improvement in peak rate compared to previously known schemes. In [6], with a limited Back-haul capacity and proactive caching exploiting context-awareness and social networks, the authors showed that the backhaul traffic load can be substantially reduced. In [7], the authors studied the performances of both deterministic and random caching in Device-to-Device networks. In [8], a hierarchical caching system with two layers of caches was considered. The authors proposed a coded caching scheme that attains the optimal memory-rate tradeoff to within a constant gap. In [9], the authors addressed the importance of the users popularity profile

estimation. The authors presented a transfer learning approach in order to enhance popularity estimates. The trade-off between collaboration distance and interference was studied in [10] for D2D networks. The authors showed that with enough content reuse, non-vanishing throughput per user can be attained, even with limited storage and delay. The impact of proactive caching on the energy efficiency was investigated in [11]. The different key factors that impact the EE of cache enabled networks were studied. The authors showed that EE can be improved by caching at the BSs when, power efficient cache hardware and sufficient cache capacity are used. Clustering users according to their request pattern was investigated in [12] with the goal of reducing service delay. The authors showed that the clustering scheme outperform the unclustered and random caching approach. In order to properly adapt to heterogeneous users behavior, we adapt a clustering based caching strategy. While in [12], a spectral clustering algorithm is used, we will use a statistical model selection approach, namely the Akaike Information Criterion, in order to efficiently estimate the number of user clusters. This will mitigate the computational complexity of eigenvalue-based algorithms, which can be prohibitive. In the proposed scheme, the clustering is performed in a global and adaptive way. The number of clusters can be estimated whenever a considerable change in user behavior was noticed. In this work, we have two objectives, namely, improving the energy efficiency and the cache hit probability. Since these two objectives may be conflicting, we choose to decouple the optimization of the two metrics. We proceed by optimizing the SBS fractions associated with each cluster in order to enhance the cache hit probability. We then optimize the achievable EE of the network with respect to the density of active small base stations. Finally, we investigate the impact of user request correlation on the performances. We propose a small base station allocation algorithm in order to cache the content closer to the users that are most likely to request it. This optimization will enable a more energy efficient scheme that exploit any spatial correlation in user traffic pattern in order to reduce the average consumed power which will, consequently, improve the achievable energy efficiency.

#### A. *Contribution and outcomes*

The main contributions of our work are presented as follows:

- 1) A clustering framework for caching: Given the heterogeneous user profiles, we propose a clustering scheme in which, users are grouped according to their popularity vectors.
- 2) Cluster estimation: The number of user clusters is not an a priori information in the network. In order to efficiently group users according to their popularity profiles, we use

the Akaike Information Criterion. This will allow to effectively estimate the number of clusters together with the most popular files in each one. This will enable the network to adapt to new users since the number of clusters will be estimated whenever a considerable change in file demand is recorded.

- 3) Cache hit probability optimization: Once the users are grouped, a given small base station will fill its cache with the most popular files of the corresponding cluster. We then optimize the cache hit probability with respect to the fractions of the small base stations caching the files of each cluster.
- 4) Energy efficiency optimization: We derive a closed form expression of the achievable energy efficiency of the network where, channel inversion power control, the power needed for the infrastructure and file fetching are taken into consideration. We then optimize the achievable energy efficiency with respect to the optimal density of active small base stations.
- 5) Optimizing Small base station allocation: After finding the optimal fractions and density of the active SBSs, we go further and select the SBS that must be allocated to each cluster. We formulate a combinatorial problem that optimizes the allocation of the small base stations to the different clusters. The problem aims at caching in each small base station the files that are most likely to be requested within their neighborhood. This optimization will, consequently, result in improving the energy efficiency of the network.

The paper is organized as follows: We describe the system model in Section II. User clustering will be investigated in Section III. In section IV, we investigate the cache hit probability. Energy efficiency will then be addressed in section V. In section VI, we present the small base station allocation algorithm. Finally, in Section VII, numerical results are presented.

## II. SYSTEM MODEL AND PRELIMINARIES

### A. Network Model

We consider a small cell network deployed over a disc with radius  $R_n$ . The small base stations are spatially distributed according to a homogeneous Poisson point process  $\phi_s$  with density  $\lambda_s$ . Each small base station can only serve one user each time with all available resources. The users are also distributed in  $\mathbb{R}^2$  according to an independent homogeneous Poisson point process  $\phi$  with density  $\lambda$  such that  $\lambda \gg \lambda_s$ . Each user is equipped with a single antenna and is allowed to communicate with any small base station within a radius  $R$ . We consider that a received packet

can be successfully transmitted and decoded if and only if  $SINR > \theta$ . This means that if the SINR is lower than certain threshold  $\theta$ , the link undergoes an outage and the transmission fails.

A general power law pathloss model is used where the power decay is given by  $r_{us}^{-\alpha}$ , where  $r_{us}$  represents the distance between user  $u$  and its serving small base station  $s$ .  $\alpha > 2$  denotes the pathloss exponent. The wireless channel from user  $u$  to the SBS  $s$  is then given by:  $g_{us} = \sqrt{r_{us}^{-\alpha}} h_{us}$  where  $h_{us}$  represents the small-scale fading coefficient modeled as Rayleigh fading i.e.,  $CN(0, 1)$  distributed random variable. We consider that the transmit power, used in both uplink and downlink, is defined according to channel inversion power control [19]. This is done so that the transmit power compensates the path-loss in order keep the average signal power received at the receiver (i.e., the small base station or the user terminal) equal to a certain constant value  $\rho_0$ . Then the power used by user  $u$  to communicate with small base station  $s$ , according to channel inversion power control, is given by:  $\rho_{us} = \rho_0 r_{us}^\alpha$ . The channel inversion power control will ensure a limitation of the interference since the power received at any base station from a typical user is upper bounded by  $\rho_0 R^\alpha$ , where  $R$  denotes the maximum communication radius.

### B. User association and Caching strategy

We consider a file catalog  $C = \{1...F\}$  containing  $F$  files with equal size of  $L$  bits. In this paper, we consider that the users have heterogeneous file popularity distributions. Each user  $u$  is associated with a popularity vector  $P_u = [p_{1u}...p_{Fu}]$ , where  $p_{iu}$  denotes the probability that user  $u$  will request file  $i$ . We consider that these probabilities change slowly over time and that they are previously known by the network. Estimating the users popularity distributions can be performed by learning from previously recorded user requests [20]. Although, users have heterogeneous popularity profiles, We suppose that they can be grouped according to there interest into  $N_c$  clusters. This means that the users from each cluster will have a certain correlation in their request patterns.

Each small base station is equipped with a caching capacity of  $M$  files. In order to guarantee content diversity in the network and to cope with the heterogeneous user profiles, the most popular content of each cluster should be cached. Each individual SBS will cache the  $M$  most popular files from a given cluster. The density of small base stations caching the popular files of cluster  $k$  is  $\lambda_{sk}$  where  $\lambda_{sk} = x_k \lambda_s$ .  $x_i$  represent the fraction of SBSs associated with cluster  $i$ ,  $i = 1..N_c$ . Each user will look for the requested file in the cache of the SBSs within a radius  $R$ . If the requested file is available in a cache within this distance, a cache hit event occurs

and the user will associate with the closest small base station storing the requested file. In the event of a cache miss, the user will simply associate with the nearest small base station from its corresponding cluster and the requested content will be retrieved from the core network through the back-haul.

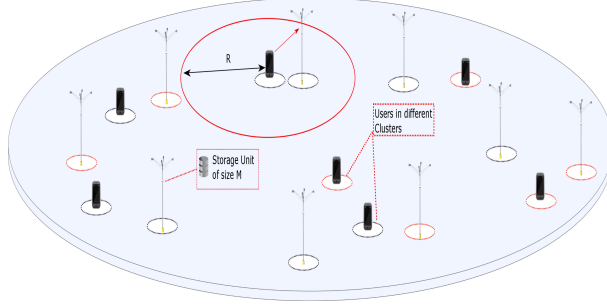


Fig. 1: System Model

### III. POPULARITY BASED USER CLUSTERING

In this section, we address user clustering based on similarities in their traffic pattern. We will use the Akaike information criterion as a tool for statistical model selection in order to efficiently estimate the number of user clusters. Additionally, it will allow us to estimate the average file popularity per cluster.

#### A. Cluster Estimation: Akaike information criterion

In the considered setting, the users have heterogeneous popularity profiles. However, the different social relations and interactions may result in some correlation in user traffic pattern. The number of clusters based on popularity vectors should be estimated since this information is unknown a priori. Allowing the system to estimate this parameter periodically or whenever a substantial change in user interest is recorded, allows the network to cope with user mobility and any modification in user request pattern. The clustering should be done so that users from the same clusters have minimum divergence in their traffic pattern. Estimating the number of clusters will require the use of a statistical model selection criterion, namely, *AKAIKE INFORMATION CRITERION* (AIC) [14]. The AIC allows to assess the quality of statistical models for a given set of data. The AIC is based on the *KULLBACK-LEIBLER* information, also known as discrepancy, which measures the loss of information when changing the statistical model. It also addresses the

trade-off between the fitness of the statistical model based on maximum likelihood estimation and its complexity which is given by the number of parameters to be estimated.

The AIC can be better understood by considering a generating model for the popularity vectors where the true number of clusters is  $N_c$  and an approximation model  $\xi_i$  characterized by  $N_i$ , then the discrepancy between the two models is given by [15]:

$$d(N_i, N_c) = \mathbb{E} \{-2 \log(L_{\xi_i})\}$$

Here  $L_{\xi_i}$  denotes the likelihood of the approximation model knowing the popularity profiles of the users (the expression will be given later on in this section). In order to evaluate this discrepancy, we need the knowledge of  $N_c$ . A biased estimate of the discrepancy is given by  $-2 \log(L_{\xi_i})$  [15]. After bias adjustment, the discrepancy can be approximated by:

$$\mathbb{E} \{d(N_i, N_c)\} \approx \mathbb{E} \{2k_i - 2\log(L_{\xi_i})\}$$

Here  $k_i$  denotes the number of characterizing parameters in model  $\xi_i$ . The AIC of each considered statistical model is then given by:

$$AIC(\xi_i) = 2k_i - 2\log(L_{\xi_i})$$

In order to approximate the process generating the users probability vectors, we consider a set of statistical models  $\Xi = \{\xi_{N_{cmin}} \dots \xi_{N_{cmax}}\}$  where  $\{N_{cmin} \dots N_{cmax}\}$  represents the range over which the search for the true number of clusters will be carried out. Each of the considered models will be typified by a number of defining parameters. In our case, each cluster will be characterized by the average and the variance of file popularity within the cluster. Each considered model  $\xi_i$  is, consequently, characterized by  $i \times (F + 1)$  parameters,  $i \times F$  representing the average file popularity in each cluster and  $i$  variance estimate representing measuring the divergence in user traffic pattern. In this paper, the likelihood  $L_{\xi_i}$  is computed based on a Gaussian Mixture model which is a common assumptions when clustering data. The likelihood function  $\log(L_{\xi_i})$  is then given by [16]:

$$\log(L_{\xi_i}) = \log\left(\prod_{u=1}^{N_u} \mathbb{P}(P_u)\right) = \sum_{u=1}^{N_u} \left( \log\left(\frac{1}{\sqrt{2\pi}\hat{\sigma}_{\phi(u)}^F}\right) - \frac{\|P_u - \hat{P}_{\phi(u)}\|^2}{2\hat{\sigma}_{\phi(u)}^2} + \log\left(\frac{N_{\phi(u)}}{N_u}\right) \right) \quad (1)$$

where  $\phi(u)$  represent the cluster to which user  $u$  is assigned.  $\hat{P}_{\phi(u)}$  denotes the average popularity vector in cluster  $\phi(u)$ . The resulting variance estimate for cluster  $k$  is given by the following:

$$\hat{\sigma}_k^2 = \frac{1}{(N_k)} \sum_{u \in k} \|P_u - \hat{P}_k\|^2 \quad (2)$$

Then the log-likelihood function can be written as:

$$\log(L_{\xi_i}) = \sum_{k=1}^i -\frac{N_k}{2}(\log(2\pi) - 1 + 2\log(\frac{N_k}{N_u}) - F\log(\hat{\sigma}_k^2)) \quad (3)$$

We then assess the discrepancy of the different considered models and select the one with minimum discrepancy:

$$\xi_{AIC} = \underset{\xi}{\operatorname{argmin}} AIC(\xi) \quad (4)$$

The AIC allows to address the trade-off between the complexity and the suitability of the model.

### B. User Clustering Algorithm

The proposed algorithm starts by assuming a search interval  $[N_{cmin} \dots N_{cmax}]$ . It will begin by assuming the existence of  $N_{cmin}$  clusters and then it will add a new cluster at each step until reaching the upper bound of the interval. The AIC is decreasing as a function of the number of clusters until reaching a minimum in the most accurate estimate. The AIC will then start increasing because of model complexity. Since the goal of the clustering is to reduce the divergence among users from the same cluster, a new centroids will be added, at each step, in the cluster with the greatest popularity variance. The new center will be selected as the user having the largest distance from the mean popularity vector of its cluster. This will allow us to reduce the discrepancy in user traffic pattern in each cluster. Once a new centroid is defined, the K-mean algorithm will be used in order to allocate users [22].

The K-mean algorithm allows to assign each user to the cluster with the nearest centroid which results in minimizing the disparity between users behaviors in the same cluster. The popularity profile of the cluster will then be defined as the average of the popularity vectors of all users in the cluster as:  $\hat{P}_k = \frac{\sum_{u, \phi(u)=k} P_u}{N_k}$ . Then the distances will be recomputed and the users will be reassigned. This procedure will be repeated until no assignment change is observed. The proposed clustering algorithm can be written as the following:



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### User Clustering Algorithm

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*Initialize:* Cluster number interval  $[N_{cmin}...N_{cmax}]$ , Set  $K = N_{cmin}$

Choose randomly the first  $N_{cmin}$  centroids from the users

1. Run  $K - mean$  algorithm and compute  $AIC(\xi_{N_{cmin}})$
  2. Choose the user having the largest distance from its centroid in the cluster with the greatest variance
  3. Add a centroid with the popularity profile of the chosen user and set  $K = K + 1$
  4. Run steps 1 and 3 until reaching  $N_{cmax}$
  5. Choose the model which minimizes the AIC and cluster the users accordingly
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### IV. CACHE HIT PROBABILITY

In order to assess the achieved performances through user clustering based on content popularity, we investigate the cache hit probability. A cache hit event occurs when a given user finds the requested content in the cache of SBS within a radius  $R$  [13]. Our context is different from the one in [13], since the users are clustered and the SBS cache different files depending on their associated cluster. Considering the proposed clustering model, the cache hit probability can be expressed as follows (the derivations are skipped for brevity):

$$\mathbb{P}\{hit\} = \frac{1}{N_u} \sum_{k=1}^{N_c} \sum_{u=1}^{N_u} \left( \sum_{i \in \Delta_k} p_{iu} \right) \left( 1 - e^{-x_k \lambda_s \pi R^2} \right) \quad (5)$$

Where  $\Delta_k$  represents the set of the  $M$  most requested files of cluster  $k$ . This equation denotes the probability of finding of at least one small base station with the requested file stored in its cache within a radius  $R$  from the user. The average number of SBS caching the most popular content of a cluster  $k$  is given by  $x_k \lambda_s \pi R^2$ .

Once all users are allocated to their respective clusters, we can improve the cache hit probability by optimizing the density of the small base stations caching the files of each cluster. We optimize the cache hit probability with respect to the fractions  $[x_1...x_{N_c}]$  where each  $x_i, i = 1..N_c$  refers to the fraction of SBS assigned to cluster  $i$ . We formulate the following optimization problem over the fraction vector  $X = [x_1...x_{N_c}]$ :

$$\begin{aligned} & \underset{X}{\text{maximize}} \quad \frac{1}{N_u} \sum_{k=1}^{N_c} \sum_{u=1}^{N_u} \left( \sum_{i \in \Delta_k} p_{iu} \right) \left( 1 - e^{-x_k \lambda_s \pi R^2} \right) \\ & \text{subject to} \quad \sum_{k=1}^{N_c} x_k \leq 1 \end{aligned} \quad (6)$$

Intuitively, the optimization of the SBS fractions should result in allocating more SBS to the Cluster with the most popular cached content, while maintaining an adequate file diversity in the network. We can show that the considered optimization problem is concave. Consequently, the optimal solution can be derived using the KKT conditions.

**lemma 1.** *The optimal fraction values are then given by:*

$$x_s = \frac{N_c \log(\psi_s) - \sum_{k=1}^{N_c} \log(\psi_k) + \lambda_s \pi R^2}{N_c \lambda_s \pi R^2}, \quad \forall s = 1..N_c \quad (7)$$

$$\text{where } \psi_s = \lambda_s \pi R^2 \sum_{u=1}^{N_u} \left( \sum_{i \in \Delta_s} p_{iu} \right)$$

*Proof.* See Appendix. A □

The optimized network will allocate, to each cluster, a number of SBSs that is proportional to the popularity of its stored files. This optimization will result in maintaining a diversity in the stored content and, more importantly, in reducing the average distance between the users and the SBSs storing the most popular files among all clusters. This will enable to reduce the used transmit power which, consequently, should improve the energy efficiency. One of the major advantages of proactive caching is energy efficiency [11]. In fact, it has the upside of increasing the network capabilities without the need for more expensive infrastructure. The Green aspect of cache enabled wireless networks was investigated in [11]. It showed that proactively caching popular content can enable a significant reduction of energy consumption in the network. In the next section, we investigate the energy efficiency of the proposed clustering framework.

## V. ENERGY EFFICIENCY

In this section we are interested in investigating the achievable performance in offloading success probability of our caching system. We then, investigate the Energy efficiency and derive the optimal SBS density that optimizes the system performances. We consider the Downlink of the cache enabled network. Now, without loss of generality we concentrate on a reference user located at the origin of the plane.

### A. Average Total Power

In order to gain a useful insight on the energy efficiency and capture the fundamental tradeoffs, we extend the power model in [11]. The average consumed total power in the considered network

with caching capabilities can be modeled as follow:

$$\rho_{total}^c = \mathbb{E}\{\rho_I\} + \mathbb{E}\{\rho_T\} + \mathbb{E}\{\rho_f\} \quad (8)$$

Where  $\rho_I, \rho_T$  and  $\rho_f$  respectively denote the power consumed by the infrastructure of active base stations, the total transmit power and the used power to fetch a file from the hard disc or the core network. The average power consumed by the infrastructure is given by:

$$\mathbb{E}\{\rho_I\} = \rho \lambda_s \pi R_n^2 \quad (9)$$

$\rho$  and  $\lambda_s \pi R_n^2$  denotes, respectively, the fix operational charge consumed by an active small base station and the average number of small base stations within the network.

The average power used to retrieve a file either over the back haul when a cache miss event occurs or from the small base station cache is given by:

$$\mathbb{E}\{\rho_f\} = \lambda_s \pi R_n^2 (\rho_{hd} \mathbb{P}\{hit\} + \rho_{bh} (1 - \mathbb{P}\{hit\})) \quad (10)$$

Here  $\rho_{hd}$  denotes the power needed to retrieve data from the local hard disk of a small base station when the requested content is already cached and a cache hit event occurs.  $\rho_{bh}$  denotes the power needed to retrieve data from the backhaul when a miss event occurs. The power used for transmission depends on the distance between the communicating small base station and user. Here we consider  $\Upsilon_k$  as the set of users associated with cluster  $k, \forall k = 1..N_c$ . Each user will associate with the nearest base station from the same cluster. A user will only communicate with base stations from another cluster only when the requested file is already cached and a hit event occurs. Taking into consideration the density of SBS associated with each cluster, and the channel inversion power control, the average total power used for transmission is given by:

$$\mathbb{E}\{\rho_T\} = \frac{\lambda_s \pi R_n^2}{N_u} \sum_{k=1}^{N_c} \sum_{u \in \Upsilon_k} \left( \mathbb{E}\{\rho_k\} + \sum_{j \neq k} \sum_{i \in \Delta_s} p_{iu} (1 - e^{-x_j \lambda_s \pi R^2}) (\mathbb{E}\{\rho_j\} - \mathbb{E}\{\rho_k\}) \right) \quad (11)$$

Here  $E\{\rho_k\}$  denotes the average transmit power used when communicating with small base stations associated with cluster  $k$ . Finally, the expression of the average consumed total power is derived by including the expressions of  $E\{\rho_k\}, \forall k = 1..N_c$ :

**lemma 2.** *The average consumed total power in the considered network with caching capabilities can be modeled as follow:*

$$\begin{aligned} \rho_{total}^c = & \lambda_s \pi R_n^2 (\rho_{hd} \mathbb{P}\{hit\} + \rho_{bh} (1 - \mathbb{P}\{hit\}) + \rho) \frac{\lambda_s \pi R_n^2}{N_u} \sum_{k=1}^{N_c} \sum_{u \in \Upsilon_k} \left( \frac{\rho_0 \gamma (\frac{\alpha}{2} + 1, \pi \lambda_{sk} R^2)}{(\lambda_{sk} \pi)^{\frac{\alpha}{2}}} \right) \\ & + \sum_{j \neq k} \sum_{i \in \Delta_s} p_{iu} (1 - e^{-x_j \lambda_s \pi R^2}) \left( \frac{\rho_0 \gamma (\frac{\alpha}{2} + 1, \pi \lambda_{sj} R^2)}{(\lambda_{sj} \pi)^{\frac{\alpha}{2}}} - \frac{\rho_0 \gamma (\frac{\alpha}{2} + 1, \pi \lambda_{sk} R^2)}{(\lambda_{sk} \pi)^{\frac{\alpha}{2}}} \right) \end{aligned} \quad (12)$$

*Proof.* See Appendix. B □

Following the same reasoning, the average consumed total power in the network with no data caching capabilities at the base stations, is given by:

$$\rho_{total}^{nc} = \lambda_s \pi R_n^2 \rho_{bh} + \rho \lambda_s \pi R_n^2 + \lambda_s \pi R_n^2 \frac{\rho_0 \gamma (\frac{\alpha}{2} + 1, \pi \lambda_s R^2)}{(\lambda_s \pi)^{\frac{\alpha}{2}}} \quad (13)$$

$\rho_{total}^{nc}$  will be used in order to investigate when enabling caching at the base station can improve the Energy efficiency of the network.

### B. Average Spectral Efficiency

The Energy Efficiency is defined as the ratio of the average achievable spectral efficiency to the average consumed power [11]. A user will connect with the nearest small base station caching the requested file from cluster  $k$  in the case of a cache hit event. The downlink SINR for a reference user  $u$  taken at the origin is:

$$SINR = \frac{\rho_0 h_u}{\sigma^2 + \sum_{k=1}^{N_c} I_{ku}}$$

where  $I_k \forall k = 1..N_c$  represents the interference coming from SBS from cluster  $k$  given by  $I_{ku} = \sum_{i \in \phi_{sk}} \rho_{ik} h_{ik} R_{ik}^{-\alpha}$ . Here  $\alpha$  denotes the path-loss exponent,  $\rho_{ik}$  the power used in the downlink by SBS  $i$  from cluster  $k$ . In order to compute the average spectral efficiency of the network, first we need to derive the achievable coverage probability. Since the aim is to guarantee a certain QoS for the users in the network, we suppose that the transmission will succeed when the downlink SINR is higher than a given threshold  $\theta$ . Then in this case the achievable coverage probability is given in the following lemma:

**lemma 3.** *The downlink coverage probability is given by:*

$$\mathbb{P}\{SINR \geq \theta\} = \exp\left(-\frac{\theta}{\rho_0} \sigma^2\right) \prod_{k=1}^{N_c} \exp\left(-\pi \lambda_{sk} \Gamma\left(1 + \frac{2}{\alpha}\right) \Gamma\left(1 - \frac{2}{\alpha}\right) \left(\frac{\theta}{\rho_0}\right)^{\frac{2}{\alpha}} \mathbb{E}\left[P_k^{\frac{2}{\alpha}}\right]\right) \quad (14)$$

*Proof.* See Appendix. C □

We can see from lemma 3, that increasing the Small Base Station density enables to reduce the used transmit power. Nevertheless, we need to take into consideration the constant power needed by the SBS even in Idle mode which represent an important part of the power consumption of the network. The average achievable spectral efficiency can be written as:

$$\xi = \lambda_s \pi R_n^2 \log(1 + \theta) \mathbb{P}\{SINR \geq \theta\} \quad (15)$$

Given the average achievable spectral efficiency and average consumed power, we can derive a closed form expression of the energy efficiency  $\Sigma$ :

$$\Sigma = \frac{\lambda_s \pi R_n^2 \log(1 + \theta) \mathbb{P}\{SINR \geq \theta\}}{P_{total}^c} \quad (16)$$

### C. Analysis of Energy Efficiency

We now concentrate on evaluating the impact of key factors, namely transmit power and small base station density, on the Energy Efficiency of the considered network. We can see from the expression of the achievable average spectral efficiency that increasing the small base station density will result in a reduction in the interference level. This is mainly due to the resulting decrease in transmit power since users are closer to their serving small base stations. Nevertheless, increasing small base station density will result in more power consumption due to the active infrastructure. We aim then at finding the optimal Small Base Station density that maximizes the achievable Energy Efficiency. We can imagine a setting in which SBS are activated and shutdown based on the user density and popularity profile. We then formulate an optimization problem that aims at maximizing the achievable energy efficiency of the network. In order to be sure that enabling caching in the network results in lower power consumption, we consider a constraint in which we aim at maintaining a power budget that is lower than that used when no caching capabilities are considered. The considered optimization problem can be formulated as follow:

$$\begin{aligned} & \underset{\lambda_s}{\text{maximize}} \quad \Sigma \\ & \text{subject to} \quad \rho_{total}^c - \rho_{total}^{nc} \leq 0 \end{aligned} \quad (17)$$

This optimization problem will allow to derive the optimal density of active small base station needed to maximize energy efficiency for a given user density, popularity profile and cache size.

**Theorem 1.** *The considered optimization problem is quasi concave and the optimal Small Base station density  $\lambda_s$  can be derived using KKT conditions.*

*Proof.* See Appendix. D □

The detailed derivation of the solution is skipped for brevity. The derivation of optimal solutions using KKT conditions has been widely utilized in the literature. While, the optimal active small base station density can be derived using the previous Theorem, further optimization can be done by optimally allocating the cache enabled small base stations to their respective clusters. We already derived the optimal SBS density that associated with each cluster, but further improvement can be made by considering the allocation of each individual SBS. In fact, caching files closer to the users that are most likely to request them will result in lower transmit power thanks to channel inversion power control, which also reduces the interference in the network resulting in higher spectral efficiency.

## VI. EXPLOITING SPATIAL CORRELATION IN USERS DEMAND

In this section, we investigate the problem of optimal content placement. While in the previous sections, we have already derived the optimal small cell fractions and density of SBSs associated with each cluster, we still need to optimize the affectation of the SBSs to the different clusters. The geographic placement of the fetched files has a important impact on the achievable performances. In fact when the probability of coverage by more than just one small base station is high enough, increasing spatial diversity of the cached files will considerably enhances hit probability [23]. In our setting, the geographical location of the cached files has an important impact on the achievable coverage probability. In fact decreasing the distance between a given user and the small base station storing its requested file results in lower transmit power which means a more energy efficient communication and less interference. The user will use less transmit power when its is served from a small base station in its neighborhood. Minimizing the average transmit power is equivalent to finding the content that is most likely to be requested in the neighboring small base station. We develop an optimization framework in which small BS and cluster association is optimized so that any spatial correlation in the user demand can be exploited. This means that, whenever users having comparable file demand are geographically located in the same area, we should allocate the nearest BS to the cluster that describes best the users behavior. In fact we may observe a certain spatial correlation in user file demand. This

may be explained by the fact that people from the same social group (living or working in the same place) are most likely to have similarities in their traffic pattern.

In order to exploit an eventual geographical correlation in user requests, we develop an integer optimization problem, where we aim at minimizing the used power over the possible SBS affectation to their corresponding clusters. Since channel inversion power control is used, minimizing the used power is equivalent to reducing the distance between the users and the SBS caching the files they are most likely to request. We consider  $\omega_{u,s} = R_{us}^{-\alpha}$  representing the path-loss between user  $u$  and the SBS  $s$ . We sort the link path-losses in decreasing order and denote by  $(s)_u$  the small base station with the  $s$ -th greatest path-loss coefficient. We assume that any given user will associate with the closest SBS from cluster  $k$  in order to retrieve a file from  $\Delta_k$ . Since less power will be used when a user is served from a small base station within its neighborhood, maximizing  $\omega_{u,s}$  is equivalent to minimizing the transmit power and the distance between the user and its serving small base station. The average number of SBS associated with each cluster  $k$  is given by  $N'_{sk} = x_k \lambda_s \pi R_n^2$ . Since the fractions  $x_k$ ,  $k = 1..N_c$  are computed solely in order to optimize the cache hit probability, it does not take into consideration the spatial repartition of the SBS and users in the network. In the case where users from the same cluster are not located within a reduced area, this may result in a high transmit power. In order to deal with this problem, we relax the constraint on the number of SBS per cluster and we replace  $N'_{sk}$  by  $N_{sk}$  where  $N_{sk} > N'_{sk}$ . Let  $Y$  be the adjacency matrix where  $y_{s,k} = 1$  if the small base station  $s$  is associated with cluster  $k$  and 0 otherwise. The considered optimization problem can be expressed as:

$$\begin{aligned}
& \underset{Y}{\text{maximize}} \sum_{k=1}^{N_c} \sum_{u=1}^{N_u} \sum_{f \in \Delta_k} p_{fu} \sum_{s=1}^{N_s} \omega_{u,(s)_u} (y_{(s)_u,k} \prod_{i=1}^{s-1} (1 - y_{(i)_u,k})) \\
& \text{subject to} \sum_{k=1}^{N_c} y_{s,k} \leq 1, \forall s = 1..N_s \\
& \sum_{s=1}^{N_s} y_{s,k} \leq N_{sk}, \forall k = 1..N_c \\
& \sum_{s=1}^{N_s} \sum_{k=1}^{N_c} y_{s,k} \leq N_s
\end{aligned} \tag{18}$$

In what follows, we show that the considered optimization problem is NP-hard. We then prove that it can be formulated as the maximization of a submodular function over matroid constraints.

We then use a greedy algorithm to solve the optimal small base station-cluster association problem which achieves at least  $\frac{1}{2}$  of the optimal value.

#### A. Computational Intractability

We start by showing the computational intractability of the considered optimization problem in (18).

**Theorem 2.** *The considered optimization problem in (18) is NP-hard.*

*Proof.* In order to show that the resulting problem is NP-hard, we consider a special case of our setting where  $N_{sk} = N \forall k = 1..N_c$  and  $N_s = N_c$ . This special case means that the fractions of SBS associated with each cluster are the same, which is the case when  $\sum_{u=1}^{N_u} \sum_{f \in \Delta_k} p_{fu} = C \forall k = 1..N_c$  and that the number of clusters is equal to the number of available SBS. In this case, the resulting optimization problem can be written as follow:

$$\begin{aligned}
 & \underset{Y}{\text{maximize}} \sum_{k=1}^{N_c} \sum_{u=1}^{N_u} \sum_{s=1}^{N_s} C \omega_{u,(s)_u} (y_{(s)_u,k} \prod_{i=1}^{s-1} (1 - y_{(i)_u,k})) \\
 & \text{subject to} \sum_{k=1}^{N_c} y_{s,k} \leq 1, \forall s = 1..N_s \\
 & \sum_{s=1}^{N_s} y_{s,k} \leq N, \forall k = 1..N_c \\
 & \sum_{s=1}^{N_s} \sum_{k=1}^{N_c} y_{s,k} \leq N_s
 \end{aligned} \tag{19}$$

In order to show NP-hardness, we will use a reduction from the following NP-hard problem.

**Weighted K-Set Packing Problem:** K-Set packing is an NP-hard combinatorial problem, it is one of the 21 problems of Karp [25]. The K-Set packing problem asks to find a maximum number of pairwise disjoint sets, with at most  $K$  elements, in a family  $S$  of subsets of a universal set  $V$ . The weighted version of the K-Set packing problem is obtained by assigning a real weight to each subset and maximizing the total weight.

We consider a collection of SBS sets  $\{v_i, i = 1, \dots, n\}$  associated each with a weight  $\omega_{v_i} =$



$\sum_{u=1}^{N_u} \max_{s \in v_i} \omega_{u,s}$ . Problem (19) can then be formulated as a Weighted K-Set Packing Problem:

$$\begin{aligned}
& \underset{X}{\text{maximize}} \sum_i C\omega_{v_i} x_i \\
& \text{subject to } v_i \cap v_j = \emptyset, \forall i, j \\
& |v_i| \leq N, \forall i = 1..N_c \\
& x_i \in \{0, 1\}
\end{aligned} \tag{20}$$

Solving (20) will result in at most  $N_c$  sets of SBSs. Since the resulting sets are disjoint, each of them will be associated with a given cluster. The number of resulting sets could not exceed  $N_c$  since  $N_s = N_c$ . We can see that solving the weighted K-Set Packing Problem, for  $K = N$  and where the weight of each subset is given by  $C\omega_{v_i}$ , is equivalent to solving the special case of the small base station allocation problem in (19).

Knowing the Weighted K-Set Packing Problem is NP-hard, we can then conclude that the considered combinatorial problem for small base station allocation in (18) is NP-hard.  $\square$

### B. Approximation

In order to solve the considered optimization problem (18), we start by showing that it is equivalent to the maximization of a Sub-modular set function over matroid constraints. This will allow the use of a greedy algorithm that provides a solution that achieves at least  $\frac{1}{2}$  of the optimum value. We provide the definitions of matroids and sub-modular set functions [17]:

**Definition 1.** Let  $S$  be a finite ground set. A submodular set function  $f : 2^S \rightarrow R$  is a function that verifies for all  $A, B \subseteq S$ :

$$f(A) + f(B) \geq f(A \cup B) + f(A \cap B).$$

*Intuitively, submodular functions capture the concept of diminishing returns.*

**Definition 2.** A matroid  $M$  is a tuple  $M = (S, I)$ , where  $S$  is a finite ground set and  $I \subseteq 2^S$  (the power set of  $S$ ) is a collection of independent sets, such that:

- $I$  is nonempty, in particular.
- $I$  is downward closed; i.e., if  $Y \in I$  and  $X \subseteq Y$ , then  $X \in I$
- If  $X, Y \in I$  and  $|X| < |Y|$  then  $\exists y \in Y \setminus X$  such that  $X \cup y \in I$

Taking into consideration the problem constraints we have the following:

**lemma 4.** *The Considered Optimization problem in (18) is equivalent to a maximization of a sub-modular set function over matroid constraints.*

*Proof.* See Appendix. E □

In order to solve the considered problem, we use the common greedy approach [18]. This algorithm starts from an empty set and, at each step, allocate a SBS to the cluster for which the added marginal value is maximized. This is done while maintaining the feasibility of the solution. We consider  $\Phi_{s,k}$  to be the set of the users that associate with SBS  $s$  to retrieve files from cluster  $k$ . At each iteration, a SBS  $s$  will be added to the cluster  $k$  with the greatest  $\sum_{u \in \Phi_{s,k}} \sum_{f \in \Delta_k} p_{fu} \omega_{u,s}$  if the constraint of the cardinality of this cluster is not violated. After  $N_s$  iteration all the small base stations will be allocated to their respective clusters. Each iteration includes the evaluation of at most  $N_s N_c$  marginal values with  $O(N_u)$  iterations for each evaluation. Then the running time of the algorithm is  $O(N_u N_s^2 N_c)$ . Running this algorithm will provide a solution that achieves at least  $\frac{1}{2}$  of the optimum value of the considered optimization problem[18].

## VII. NUMERICAL RESULTS AND DISCUSSION

In this section, we start by validating our analysis and then investigate the impact of the different system parameters on the Cache hit probability and Energy efficiency. We then investigate the impact of the small base station allocation algorithm on the performances of the network. We consider a circular region with radius  $R_n = 6Km^2$ . We simulate two PPP processes, one for the users and another for the SBSs over this area. The respective densities of these process are  $\lambda$  and  $\lambda_s$  with  $\lambda \gg \lambda_s$ . We consider a catalog constituted of  $F = 2000$  files with fixed size  $L = 20MB$ . We also consider the normalized cache size  $\eta = \frac{M}{F}$ . We characterize each cluster by a given set of popular files. We then generate the popularity profiles of the users  $P_u = [p_{1u} \dots p_{Fu}]$  randomly with a bias for one of the file sets. This will result in a random allocation of the users to the different clusters. In order to run the Clustering algorithm we only need an interval over which the search of the number of clusters is made. In our simulations we take  $[N_{cmin} \dots N_{cmax}] = [5 \dots 30]$ . We consider the pathloss exponent  $\alpha = 2.5$ . In order to investigate the achievable Energy efficiency and coverage probability, we consider the following values for the different powers[11]:

$\rho_o(dBm)$	21	$\rho_{bh}(W)$	10W
$\rho(W)$	10.16	$\rho_{hd}(W)$	$12.5 \times 10^{-5}$

Here we considered a microwave backhaul link and a speed SSD for the cache. The noise power is set to  $\sigma^2 = 95dBm$ . The simulation is repeated 10000 times.

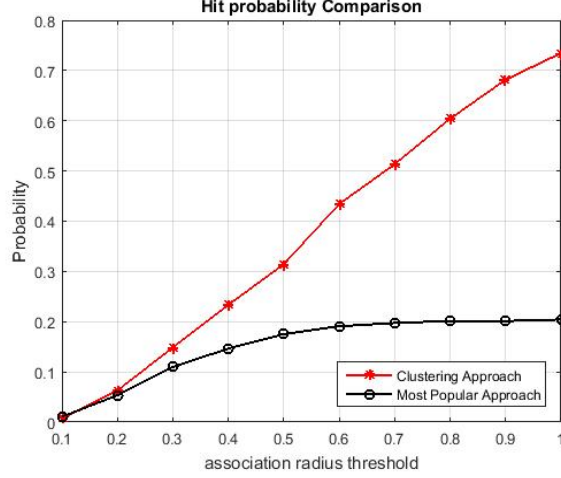


Fig. 2: Hit probability versus communication radius

Figure 2 shows the gain in cache hit probability when comparing the proposed clustering framework and the scheme in which the most popular files among all users are cached in all cells. For a communication radius of 0.9 Km, we notice a substantial gain with a cache hit probability of 0.89 for the clustering scheme compared with a probability equal to 0.2 when caching the most popular files in all SBSs. Figure 3 shows the changes in the cache hit probability for different SBS densities. We notice that the hit probability for the scheme without clustering saturates at a low value. This is due to the fact that the same set of files is cached in all the SBSs which is clearly a suboptimal approach, especially when users are covered by multiple SBSs. The increase in hit probability for the clustering method is mainly due to the diversity of files cached in the SBS. Increasing the SBS density results in reducing the average distance from mobile users, which, consequently, results in improving the cache hit probability.

Figure 4 shows the performance of AIC based model selection. The figure represents the computed AIC per point over the estimation range. The lowest AIC value represents the model that strikes the best trade-off between fitness and complexity. The negative values of the AIC are due to a negative bias which characterize the AIC when dealing with a small sample number.

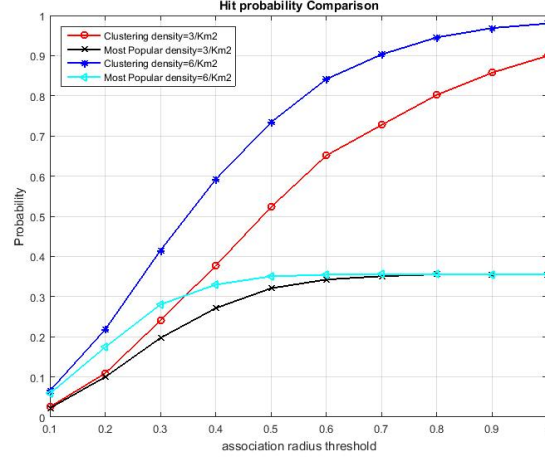


Fig. 3: Hit Probability with different SBS densities

Figure 5 validate our analysis in section VI and show that the derived expression accurately capture the achievable coverage probability in the network. In Figure 6 we show the numerical

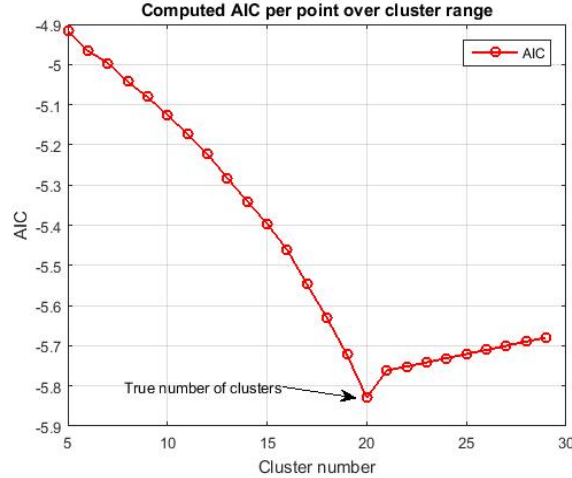


Fig. 4: Akaike information criterion

results of energy efficiency versus the SBS density and the normalized cache capacity. With a given cache capacity, the EE has a quasi-concave behavior as a function of the SBS density. We can see that the SBS density has a huge impact on the EE. This is mainly due to the infrastructure power consumption. Although increasing the SBS density reduces the average transmit power, we see that this impact is minor compared with the energy expenses related to the active infrastructure. Increasing the normalized cache capacity results in an improvement in

the EE efficiency. This means that increasing the cache size in each SBS is a very practical way to improve the system performances without the need for costly infrastructure.

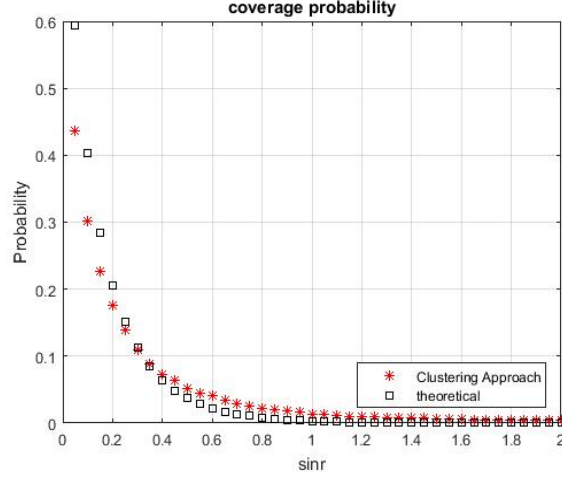


Fig. 5: Coverage probability

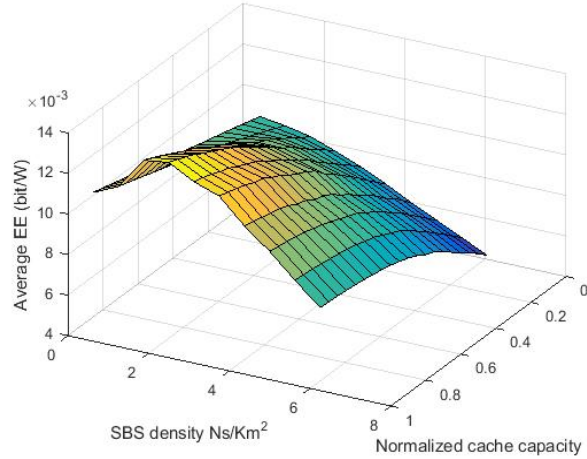


Fig. 6: Energy Efficiency vs normalized cache size and SBS density

Figure 7 shows the impact of the SBS allocation algorithm from section VII on the EE. We can clearly see that, for different values of the SBS density, optimizing the SBS allocation to the different clusters results in a considerable gain in the EE. For a SBS density of  $\lambda_s = 1.9$  and a normalized cache size of 0.4, optimizing the allocation of the SBS result in a gain of 36.6% in the EE. As the density of SBS increases the allocation algorithm will result in greater

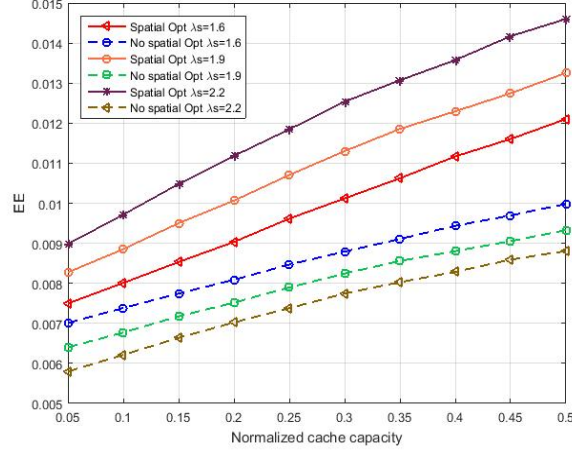


Fig. 7: EE versus normalized cache size: impact of SBS allocation algorithm

improvement in EE. Optimizing the cluster SBS association will result in less average transmit power, which, consequently, reduces the interference in the network. The algorithm will result in reducing the average distance between the users and the SBS that cache the files that they are most likely to request.

In Figure 8, we plot the variation of the cache hit probability as a function of the normalized

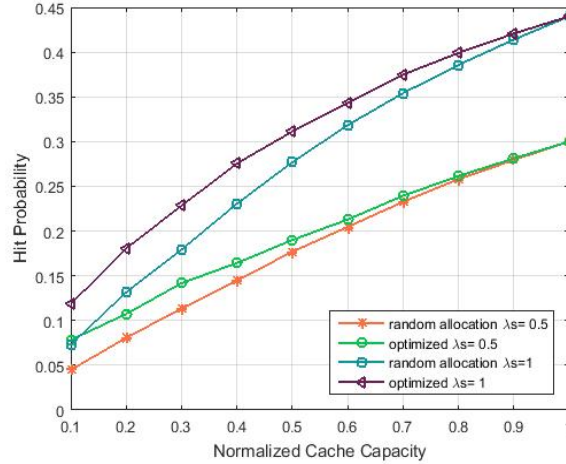


Fig. 8: Hit probability versus normalized cache size: impact of SBS allocation algorithm

cache size for two different SBS densities after applying the SBS allocation algorithm from section VII. While the proposed algorithm aims mainly at optimizing the EE, we can see that it does not degrade cache hit probability but actually enables to improve it. In fact, the allocation

algorithm tends to reduce the average distance between the users and the SBS that are caching the files that are most likely to be requested in order to decrease the transmission power which, consequently, increases the hit probability. The figure shows that, the impact of the combinatorial small base station allocation algorithm on the cache hit probability varies depending on the SBS density. In fact with more SBS in the network the optimized allocation considerably improves the cache hit probability. With a normalized cache size 0.5, the SBS allocation algorithm enables a gain of approximately 14% in the cache hit probability compared with the random SBS allocation case for a SBS density of 1 per  $Km^2$ . In the case of a SBS density of 0.5 per  $Km^2$ , this gain is almost 7.5%. The figure shows also that as the normalized cache size increases, optimizing the SBS allocation results in a decreasing gain until reaching zero for  $\eta = 1$ . In fact as the cache size increases, the impact of optimizing the SBS allocation on the cache hit probability becomes less significant since file diversity increased considerably in the users neighborhood.

## VIII. CONCLUSION

In this paper, we studied a cache enabled small cell network. A clustering approach was considered in order to exploit the correlation between users file requests. We develop an algorithm that enables the estimation of the optimal number of clusters and to assign users efficiently to their respective clusters. By considering the distribution of SBSs to be a poisson point process, we derived the hit probability when the SBS fill their cache with the most popular content of the different clusters. We also investigated the impact of caching on the Energy Efficiency of the network and proposed a small base station allocation algorithm that aims at minimizing the transmit power by bringing the cached files closer to the users that are most likely to request them. Finally, we performed numerical analysis which shows that the proposed clustering framework considerably outperforms the scheme in which the most popular files are cached in all the base stations. It also shows that optimized small base station allocation results in an improvement in the network key performance indicators such as hit probability and especially Energy Efficiency.

## IX. APPENDIX

### **Appendix A. proof of Lemma 1:**

The optimal solution of the considered optimization problem can be found using KKT conditions.

The Lagrangian associated with this problem is given by:

$$L(X, \mu) = \frac{1}{N_u} \sum_{k=1}^{N_c} \sum_{u=1}^{N_u} \left( \sum_{i \in \Delta_k} p_{iu} \right) \left( 1 - e^{-x_k \lambda_s \pi R^2} \right) - \mu \left( \sum_{k=1}^{N_c} x_k - 1 \right)$$

where  $\mu$  is the Lagrange multiplier. Taking the gradient of the Lagrangian we have the following:

$$e^{-x_k \lambda_s \pi R^2} = \frac{N_u \mu}{\lambda_s \pi R^2 \sum_{u=1}^{N_u} \left( \sum_{i \in \Delta_s} p_{iu} \right)}, \quad \forall s = 1..N_c \implies$$

$$\lambda_s \pi R^2 = \sum_{k=1}^{N_c} \log \left( \lambda_s \pi R^2 \sum_{u=1}^{N_u} \left( \sum_{i \in \Delta_k} p_{iu} \right) \right) - \log(N_u \mu) N_c$$

The fraction are then given by:

$$x_s = \frac{\log \left( \lambda_s \pi R^2 \sum_{u=1}^{N_u} \left( \sum_{i \in \Delta_s} p_{iu} \right) \right) - \log(N_u \mu)}{\lambda_s \pi R^2} = \frac{N_c \log(\psi_s) - \sum_{k=1}^{N_c} \log(\psi_k) + \lambda_s \pi R^2}{N_c \lambda_s \pi R^2}$$

where:  $\psi_s = \lambda_s \pi R^2 \sum_{u=1}^{N_u} \left( \sum_{i \in \Delta_s} p_{iu} \right)$

## Appendix B. proof of Lemma 2:

We derive the expression of the average consumed power in the network with cache enabled small base stations. The average total power  $\rho_{total}^c$  is given by:

$$\rho_{total}^c = \mathbb{E} \{ \rho_I \} + \mathbb{E} \{ \rho_T \} + \mathbb{E} \{ \rho_f \}$$

Where  $\mathbb{E} \{ \rho_I \} = \rho \lambda_s \pi R_n^2$  and  $\mathbb{E} \{ \rho_f \} = \lambda_s \pi R_n^2 (\rho_{hd} \mathbb{P} \{ hit \} + \rho_{bh} (1 - \mathbb{P} \{ hit \}))$ . Due to the channel inversion power control and the user association procedure, the average consumed transmit power is given by:

$$\mathbb{E} \{ \rho_T \} = \frac{\lambda_s \pi R_n^2}{N_u} \sum_{k=1}^{N_c} \sum_{u \in \Upsilon_k} \left( \mathbb{E} \{ \rho_k \} + \sum_{j \neq k} \sum_{i \in \Delta_s} p_{iu} (1 - e^{-x_j \lambda_s \pi R^2}) (\mathbb{E} \{ \rho_j \} - \mathbb{E} \{ \rho_k \}) \right)$$

We need then to compute the average power used by the users to communicate with the closest small base station in the case of cache miss and with the nearest small base station from a given cluster  $k$ ,  $k = 1..N_c$ . According to the PPP assumption for the location of the small base stations, the distance from a user to its nearest SBS from cluster  $k$   $r_k$  has the following *pdf*  $f_{r_k}(r) = 2\pi \lambda_{sk} r e^{-\lambda_{sk} \pi r^2}$  [19]. The transmit power used by the user in this case is given by  $\rho_k = \rho_0 r_k^\alpha$ . Then:

$$\mathbb{E} [\rho_k] = \int_0^R 2\pi \lambda_{sk} r^{\alpha+1} \exp(-\lambda_{sk} \pi r^2) dr = \frac{\rho_0 \gamma(\frac{\alpha}{2} + 1, \pi \lambda_{sk} R^2)}{(\lambda_{sk} \pi)^{\frac{\alpha}{2}}}$$



Following the same calculus for  $\mathbb{E}[\rho_k]$ ,  $k = 1..N_c$  with  $\lambda_{sk} = x_k \lambda_s$ , we obtain the final expression of the average consumed power in the network:

$$\begin{aligned} \rho_{total}^c &= \lambda_s \pi R_n^2 (\rho_{hd} \mathbb{P}\{hit\} + \rho_{bh} (1 - \mathbb{P}\{hit\}) + \rho) \frac{\lambda_s \pi R_n^2}{N_u} \sum_{k=1}^{N_c} \sum_{u \in \Upsilon_k} \left( \frac{\rho_0 \gamma(\frac{\alpha}{2} + 1, \pi \lambda_{sk} R^2)}{(\lambda_{sk} \pi)^{\frac{\alpha}{2}}} \right. \\ &\quad \left. + \sum_{j \neq k} \sum_{i \in \Delta_s} p_{iu} (1 - e^{-x_j \lambda_s \pi R^2}) \left( \frac{\rho_0 \gamma(\frac{\alpha}{2} + 1, \pi \lambda_{sj} R^2)}{(\lambda_{sj} \pi)^{\frac{\alpha}{2}}} - \frac{\rho_0 \gamma(\frac{\alpha}{2} + 1, \pi \lambda_{sk} R^2)}{(\lambda_{sk} \pi)^{\frac{\alpha}{2}}} \right) \right) \end{aligned}$$

### Appendix C. proof of Lemma 3:

We derive the achievable coverage probability when using channel inversion power control:

$$\begin{aligned} \mathbb{P}\{SINR \geq \theta\} &= \mathbb{E} \left[ \mathbb{P}(h_u \geq (\frac{\sigma^2 + \sum_{k=1}^{N_c} I_k}{\rho_0}) \theta) | I_k \forall k \right] = \mathbb{E} \left[ \exp(-\frac{\theta}{\rho_0} (\sigma^2 + \sum_{k=1}^{N_c} I_k)) | I_k \forall k \right] \\ &= \exp(-\frac{\theta}{\rho_0} \sigma^2) \prod_{k=1}^{N_c} \mathcal{L}_{I_k}(\frac{\theta}{\rho_0}) \end{aligned}$$

Where we used the fact that  $h_u$  is exponentially distributed and  $\mathcal{L}_{I_k}(s)$  is the Laplace transform of the random variable  $I_k$  at  $s$ . Then to prove the lemma we need to compute the Laplace transform of  $I_k \forall k$ .

The interfering base stations constitute multiple PPP processes  $\phi_k$  each associated with a given cluster. Then the Laplace transform of  $I_k$  for a given  $k$  is obtained as:

$$\begin{aligned} \mathcal{L}_{I_k}(\frac{\theta}{\rho_0}) &= \mathbb{E} \left[ \exp \left( - \sum_{s_i \in \phi_k} \rho_{ks_i} h_{s_i} r_{s_i}^{-\alpha} \right) \right] = \exp \left( -2\pi \lambda_{sk} \int_0^\infty \left( 1 - \mathbb{E} \left[ e^{-\theta h \rho_k r^{-\alpha}} \right] \right) r dr \right) \\ &= \exp \left( -\pi \lambda_{sk} \left( \frac{\theta}{\rho_0} \right)^{\frac{2}{\alpha}} \mathbb{E} \left[ \rho_k^{\frac{2}{\alpha}} \right] \Gamma(1 + \frac{2}{\alpha}) \Gamma(1 - \frac{2}{\alpha}) \right) \end{aligned}$$

$\mathbb{E} \left[ \rho_k^{\frac{2}{\alpha}} \right]$  depends on the density of the Small Cells caching files from cluster  $k$ . The more Small Cells associated with cluster  $k$  the less average power users utilize to communicate with this group of SBS. We can deduce from **Appendix B** that:

$$\mathbb{E} \left[ \rho_k^{\frac{2}{\alpha}} \right] = \frac{\rho_0^{\frac{2}{\alpha}} \gamma(2, \pi \lambda_{sk} R^2)}{\lambda_{sk} \pi}$$

Where  $\lambda_{sk} = x_k \lambda_s$  represent the density of SBS communication with SBS from cluster  $k$ . Which proves the Lemma. Based on Slivnyak's theorem for Poisson Point Processes [19], the obtained Expression is valid for any user within the network.

### Appendix D. proof of Theorem 1:

In order to prove Theorem 1, we need to start by showing that the objective function is quasi

concave. The objective function  $\Sigma(\lambda_s)$  can be written as a product of two nonnegative functions  $U(\lambda_s) = \lambda_s \pi R_n^2 \log(1 + \theta) \mathbb{P}\{SINR \geq \theta\}$  and  $V(\lambda_s) = \frac{1}{P_{total}^c}$ . We start by computing the derivatives of  $U(\lambda_s)$  and  $V(\lambda_s)$ :

$$U' = \frac{dU(\lambda_s)}{d\lambda_s} = \pi R_n^2 \log(1 + \theta) \exp(-\frac{\theta}{\rho_0} \sigma^2) \prod_{k=1}^{N_c} \mathcal{L}_{I_{ku}}(\frac{\theta}{\rho_0})$$

$$\left(1 - \sum_{k=1}^{N_c} \Gamma(1 + \frac{2}{\alpha}) \Gamma(1 - \frac{2}{\alpha}) \theta^{\frac{2}{\alpha}} \pi \lambda_{sk} R^2 e^{-\lambda_{sk} \pi R^2}\right)$$

Then  $\frac{dU(\lambda_s)}{d\lambda_s} > 0$  which means that  $U$  is an increasing function of  $\lambda_s$ . We do the same to  $V(\lambda_s) = \frac{1}{P_{total}^c}$ . We have:

$$V' = \frac{dV(\lambda_s)}{d\lambda_s} = \frac{-\chi}{P_{total}^c}$$

Where  $\chi$  is given by:

$$\chi = \pi R_n^2 ((\rho_{hd} - \rho_{bh}) \mathbb{P}\{hit\} + \rho + \rho_{bh}) + \frac{\pi R_n^2}{N_u} (\chi_1 + \lambda_s \chi_2)$$

$$+ \lambda_s \pi R_n^2 \frac{1}{N_u} \sum_{u=1}^{N_u} \sum_{k=1}^{N_c} \left( \sum_{i \in \Delta_k} p_{iu} \right) x_k \pi R^2 e^{-x_k \lambda_s \pi R^2}$$

Here  $\chi_1$  and  $\chi_2$  are respectively given by:

$$\chi_1 = \sum_{k=1}^{N_c} \sum_{u \in \Upsilon_k} \left( \frac{\rho_0 \gamma(\frac{\alpha}{2} + 1, \pi \lambda_{sk} R^2)}{(\lambda_{sk} \pi)^{\frac{\alpha}{2}}} + \sum_{j \neq k} \sum_{i \in \Delta_s} p_{iu} (1 - e^{-x_j \lambda_s \pi R^2}) \right)$$

$$\left( \frac{\rho_0 \gamma(\frac{\alpha}{2} + 1, \pi \lambda_{sj} R^2)}{(\lambda_{sj} \pi)^{\frac{\alpha}{2}}} - \frac{\rho_0 \gamma(\frac{\alpha}{2} + 1, \pi \lambda_{sk} R^2)}{(\lambda_{sk} \pi)^{\frac{\alpha}{2}}} \right)$$

and:

$$\chi_2 = \sum_{k=1}^{N_c} \sum_{u \in \Upsilon_k} \left( \rho_0 (R^\alpha e^{-\lambda_{sk} \pi R^2} - \frac{\gamma(\frac{\alpha}{2} + 1, \pi \lambda_{sk} R^2)^{\frac{\alpha x_k \pi}{2}} (\lambda_{sk} \pi)^{\frac{\alpha}{2} - 1}}{(\lambda_{sk} \pi)^\alpha}) + \sum_{j \neq k} \sum_{i \in \Delta_s} p_{iu} (1 - e^{-x_j \lambda_s \pi R^2}) \right)$$

$$\rho_0 \left( (R^\alpha e^{-\lambda_{sj} \pi R^2} - \frac{\gamma(\frac{\alpha}{2} + 1, \pi \lambda_{sj} R^2)^{\frac{\alpha x_j \pi}{2}} (\lambda_{sj} \pi)^{\frac{\alpha}{2} - 1}}{(\lambda_{sj} \pi)^\alpha}) + \frac{\gamma(\frac{\alpha}{2} + 1, \pi \lambda_{sk} R^2)^{\frac{\alpha x_k \pi}{2}} (\lambda_{sk} \pi)^{\frac{\alpha}{2} - 1}}{(\lambda_{sk} \pi)^\alpha} \right.$$

$$\left. - R^\alpha e^{-\lambda_{sk} \pi R^2} \right) + \sum_{j \neq k} \sum_{i \in \Delta_s} p_{iu} x_j \pi R^2 e^{-x_j \lambda_s \pi R^2} \rho_0 \left( \frac{\gamma(\frac{\alpha}{2} + 1, \pi \lambda_{sj} R^2)}{(\lambda_{sj} \pi)^{\frac{\alpha}{2}}} - \frac{\gamma(\frac{\alpha}{2} + 1, \pi \lambda_{sk} R^2)}{(\lambda_{sk} \pi)^{\frac{\alpha}{2}}} \right)$$

Then  $\frac{dV(\lambda_s)}{d\lambda_s} < 0$ . In what follows we will distinguish two cases depending on the existence of a point  $\lambda_s^*$  such that  $\nabla \Sigma(\lambda_s^*) = 0$ . If  $\exists \lambda_s^*$  such that  $\nabla \Sigma(\lambda_s^*) = 0$  then:

$$\nabla \Sigma(\lambda_s^*) = 0 \Leftrightarrow U'V + V'U = 0 \Leftrightarrow \frac{-U'V}{V'U} = 1$$

We compute the derivative of  $L(\lambda_s) = \frac{-U'V}{V'U}$  with respect to  $\lambda_s$ . The expression of the derivative is omitted here for brevity. We find that  $L'(\lambda_s) > 0$ . Since  $L(\lambda_s)$  is a strictly increasing function then, according the Theorem of intermediate value, if  $\exists \lambda_s^*$  such that  $L(\lambda_s^*) = 1$  then this point is unique. Finally, depending on the existence of  $\lambda_s^*$ , we will have two cases:

If  $\exists \lambda_s^*$  such that  $L(\lambda_s^*) = 1$  then this point is unique and  $\Sigma$  is increasing for  $\lambda_s < \lambda_s^*$  and decreasing for  $\lambda_s > \lambda_s^*$ . If, on the other hand,  $\lambda_s^*$  does not exists, then  $\Sigma$  is a strictly monotone function. This proves that the objective function  $\Sigma$  is quasi concave.

In the second step of the proof, we need to show that the constraint is also quasi concave. This will be done in a similar way as in the first step by computing its derivative. We consider  $g(\lambda_s)$  and  $h(\lambda_s)$ , given respectively by:

$$g(\lambda_s) = \lambda_s \pi R_n^2 \mathbb{P}\{hit\} (\rho_{hd} - \rho_{bh})$$

$$h(\lambda_s) = \frac{\lambda_s \pi R_n^2}{N_u} \sum_{k=1}^{N_c} \sum_{u \in \Upsilon_k} \left( \frac{\rho_0 \gamma(\frac{\alpha}{2} + 1, \pi \lambda_{sk} R^2)}{(\lambda_{sk} \pi)^{\frac{\alpha}{2}}} + \sum_{j \neq k} \sum_{i \in \Delta_s} p_{iu} (1 - e^{-x_j \lambda_s \pi R^2}) \right.$$

$$\left. \left( \frac{\rho_0 \gamma(\frac{\alpha}{2} + 1, \pi \lambda_{sj} R^2)}{(\lambda_{sj} \pi)^{\frac{\alpha}{2}}} - \frac{\rho_0 \gamma(\frac{\alpha}{2} + 1, \pi \lambda_{sk} R^2)}{(\lambda_{sk} \pi)^{\frac{\alpha}{2}}} \right) \right) - \lambda_s \pi R_n^2 \frac{\rho_0 \gamma(\frac{\alpha}{2} + 1, \pi \lambda_s R^2)}{(\lambda_s \pi)^{\frac{\alpha}{2}}}$$

Then:

$$\frac{d(\rho_{\text{total}}^c - \rho_{\text{total}}^{\text{nc}})}{d\lambda_s} = \frac{d(g(\lambda_s))}{d\lambda_s} + \frac{d(h(\lambda_s))}{d\lambda_s}$$

$\frac{d(g(\lambda_s))}{d\lambda_s}$  and  $\frac{d(h(\lambda_s))}{d\lambda_s}$  are, respectively, given by:

$$\frac{d(g(\lambda_s))}{d\lambda_s} = \pi R_n^2 (\rho_{hd} - \rho_{bh}) \left( \mathbb{P}\{hit\} + \frac{\lambda_s}{N_u} \sum_{u=1}^{N_u} \sum_{k=1}^{N_c} \left( \sum_{i \in \Delta_k} p_{iu} \right) x_k \pi R^2 e^{-x_k \lambda_s \pi R^2} \right)$$

, and,

$$\frac{d(h(\lambda_s))}{d\lambda_s} = \frac{\pi R_n^2}{N_u} \left( \frac{h(\lambda_s)}{\lambda_s} + \lambda_s (\chi_3 + \chi_4) \right)$$

Where  $\chi_3$  and  $\chi_4$  are, respectively, given by:

$$\chi_3 = \sum_{k=1}^{N_c} \sum_{u \in \Upsilon_k} \rho_0 (R^\alpha e^{-\lambda_{sk} \pi R^2} - \frac{\gamma(\frac{\alpha}{2} + 1, \pi \lambda_{sk} R^2)^{\frac{\alpha x_k \pi}{2}} (\lambda_{sk} \pi)^{\frac{\alpha}{2}-1}}{(\lambda_{sk} \pi)^\alpha})$$

, and,

$$\chi_4 = \sum_{k=1}^{N_c} \sum_{u \in \Upsilon_k} \sum_{j \neq k} \sum_{i \in \Delta_s} p_{iu} \left( \left( \frac{\rho_0 \gamma(\frac{\alpha}{2} + 1, \pi \lambda_{sj} R^2)}{(\lambda_{sj} \pi)^{\frac{\alpha}{2}}} - \frac{\rho_0 \gamma(\frac{\alpha}{2} + 1, \pi \lambda_{sk} R^2)}{(\lambda_{sk} \pi)^{\frac{\alpha}{2}}} \right) x_j \pi R^2 e^{-x_j \lambda_s \pi R^2} \right.$$

$$\left. + (1 - e^{-x_j \lambda_s \pi R^2}) \left( \rho_0 (R^\alpha e^{-\lambda_{sj} \pi R^2} - \frac{\gamma(\frac{\alpha}{2} + 1, \pi \lambda_{sj} R^2)^{\frac{\alpha x_j \pi}{2}} (\lambda_{sj} \pi)^{\frac{\alpha}{2}-1}}{(\lambda_{sj} \pi)^\alpha}) \right. \right.$$

$$\left. \left. - \rho_0 (R^\alpha e^{-\lambda_{sk} \pi R^2} - \frac{\gamma(\frac{\alpha}{2} + 1, \pi \lambda_{sk} R^2)^{\frac{\alpha x_k \pi}{2}} (\lambda_{sk} \pi)^{\frac{\alpha}{2}-1}}{(\lambda_{sk} \pi)^\alpha}) \right) \right)$$

Then the constraint is also quasi concave since  $\frac{d(g(\lambda_s))}{d\lambda_s} + \frac{d(h(\lambda_s))}{d\lambda_s} < 0$ .

In the final step of the prove, we need to show that the KKT conditions are Necessary and Sufficient to solve (17). If  $\exists \lambda_s^*$  such that  $L(\lambda_s^*) = 1$  and  $\lambda_s^*$  satisfies the constraint then,  $\lambda_s^*$  is a global optimum of (17). If  $\lambda_s^*$  does not exists or  $\lambda_s^*$  does not satisfy the constraint then, using the Necessity and Sufficiency theorems from [21] on quasi concave optimization, we can derive the optimal SBS density using KKT conditions. In fact, since  $\frac{d(\rho_{\text{total}}^c - \rho_{\text{total}}^{\text{nc}})}{d\lambda_s} \neq 0$  then, according to the Necessity Theorem of Arrow-Enthoven, any solution of the optimization problem (17) satisfies the KKT conditions. In addition, since  $\nabla \Sigma(\lambda_s) \neq 0$  over the interval defined by the constraint then, according to the Sufficiency Theorem in [21], the optimal solution of (17) can be derived using the KKT conditions.

#### Appendix E. proof of lemma 4:

First we need to prove that the objective function  $\Omega$  is sub-modular. we consider two allocations  $X$  and  $Y$  such that  $X \subseteq Y$  and we need to prove that the marginal value of adding a new allocated SBS  $l$  to cluster  $i$  in  $X$  and  $Y$  verifies:

$$\Omega(X \cup \{y_{li}\}) - \Omega(X) \geq \Omega(Y \cup \{y_{li}\}) - \Omega(Y)$$

Monotonicity is trivial since any new small base station allocation cannot decrease the value of the objective function. In order to show submodularity of the function, we will compare the marginal values of adding  $y_{li}$  to  $X$  and  $Y$ . Here we consider  $\psi_i(X \cup \{y_{li}\})$  referring to the users that change their serving small base station from cluster  $i$ . A user changes its serving small base station when the new allocated one is closer which induces less transmit power. Then:

$$\Omega(X \cup \{y_{li}\}) - \Omega(X) = \sum_{u \in \psi_i(X \cup \{y_{li}\})} \left( \sum_{f \in \Delta_i} p_{fu} \right) \left( \omega_{u\mu(u,i)}^{(X \cup \{y_{li}\})} - \omega_{u\mu(u,i)}^{(X)} \right)$$

The marginal value of adding  $y_{li}$  to  $Y$  is given by:

$$\Omega(Y \cup \{y_{li}\}) - \Omega(Y) = \sum_{u \in \psi_i(Y \cup \{y_{li}\})} \left( \sum_{f \in \Delta_i} p_{fu} \right) \left( \omega_{u\mu(u,i)}^{(Y \cup \{y_{li}\})} - \omega_{u\mu(u,i)}^{(Y)} \right)$$

Since  $X \subseteq Y$  we can deduce that  $\psi_i(Y \cup \{y_{li}\}) \subseteq \psi_i(X \cup \{y_{li}\})$ . Since a user changes its serving small base station only when a closer allocated one is available then  $\omega_{u\mu(u,i)}^{(Y \cup \{y_{li}\})} - \omega_{u\mu(u,i)}^{(Y)} > 0$  which proves that:

$$\Omega(X \cup \{y_{li}\}) - \Omega(X) \geq \Omega(Y \cup \{y_{li}\}) - \Omega(Y)$$

Then  $\Omega$  is sub-modular. The constraint  $\sum_{s=1}^{N_s} y_{sk} \leq N_{sk}, \forall k = 1..N_c$  can be rewritten as follow:

$$\{Y : |Y \cap S_k| \leq N_{sk}, \forall k = 1..N_c\}$$

which is equivalent to a matroid constraint. Then the considered optimization problem is equivalent to maximizing a sub-modular function subject to matroid constraints.

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